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Cycles Having the Same Modularity and Removable Edges in 2-Connected Graphs

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Abstract

In this paper, we consider 2-connected multigraphs in which every cycle has length congruent to a modulo b ($b \geq 2$). We prove that there exists such a graph which is homeomorphic to a simple graph with minimum degree at least three only if $a = 0$, and that there exists such a graph which is also a simple graph only if $a = 0$ and $b = 2$. We also study the distribution of paths whose internal vertices

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have degree exactly two, and show a relation between these paths and edges in a 2-connected graph whose deletion results in a 2-connected graph.

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1 Introduction

In this paper, the term "multigraph" is used for graphs which possibly have multiple edges (but not loops), and the term "simple graphs" and "graphs" for graphs which have neither multiple edges nor loops.

In almost every textbook of graph theory, we find a characterization of the bipartite graphs in terms of cycle parity.

Theorem A *A graph G is a bipartite graph if and only if every cycle in G has even length.*

On the other hand, the graphs in which every cycle has odd length form an uninteresting class. It is not difficult to see that they are graphs in which every block is either K_2 or an odd cycle. This observation leads us to the following problem.

Problem 1 Determine a pair of integers (a, b) with $b \geq 2$ and $0 \leq a < b$ such that graphs in which every cycle has length congruent to a modulo b form an "interesting" class.

The solution of this problem depends on the interpretation of "interesting", and hence is ambiguous. Therefore, we first discuss it. First of all, when we consider Problem 1, we may restrict ourselves to 2-connected graphs. If a graph is not 2-connected, we can consider the same problem in each block.

When we deal with this problem, we can ignore vertices of degree two by considering a "weighted version" of the problem. Suppose $P = x_0x_1 \dots x_l$ is a path in a 2-connected graph G with $\deg_G x_0 \geq 3$, $\deg_G x_l \geq 3$ and $\deg_G x_i = 2$ $(1 \leq i \leq l-1)$. If a cycle C contains one edge in P, it must contain P as a subpath of C. Now replace each such path by an edge x_0x_l and assign l (the length of P) to x_0x_l as its weight. The multigraph G' obtained by this conversion is 2-connected and its minimum degree is at least three, and each edge e has an integer weight. By this conversion, the problem becomes a weighted version. Let G be a multigraph and let $f\colon E(G) \to \boldsymbol{N}$. For a subgraph H of G, we define the weight of H by $\sum_{e\in E(H)} f(e)$, and denote it by $f(H)$. Now we consider the multigraphs in which $f(C) \equiv a \pmod{b}$ holds for each cycle C of G. Furthermore, since we only consider modularity, we may assume that the weight $f(e)$ has a value in $\boldsymbol{Z}/b\boldsymbol{Z}$.

Now we formulate Problem 1. For a multigraph G and a function $f\colon E(G) \to \boldsymbol{Z}/b\boldsymbol{Z}$, the pair (G, f) is called a *weighted multigraph*. If G is a simple graph, we call it a *weighted simple graph*. If f is a constant function which takes the value $f(e) = c$ for each $e \in E(G)$, we write (G, c) instead of (G, f). For example, $(G, 0)$ and $(G, 1)$ are weighted multigraphs in which each edge has weight zero and one, respectively.

For integers a and b with $b \geq 2$ and $0 \leq a < b$, let $\mathfrak{C}(a, b)$ denote the class of 2-connected weighted multigraphs (G, f) with minimum degree at least three and $f\colon E(G) \to \boldsymbol{Z}/b\boldsymbol{Z}$ such that $f(C) = a$ holds for every cycle C. The class $\mathfrak{C}(a, b)$ can be a large class or can be empty. For example, $\mathfrak{C}(0, 2)$ contains every simple 2-connected bipartite graph $(G, 1)$ with minimum degree at least three and weight one. In the other extreme, $\mathfrak{C}(1, 2)$ is an empty class as mentioned before.

Observing the above, in this paper, we set three levels for the interpretation of "interesting", and consider the following problem.

Problem 2 Determine for what values of (a, b) $\mathfrak{C}(a, b)$ becomes

(1) nonempty class,

(2) a class which contains a weighted simple graph, and

(3) a class which contains a weighted simple graph of the form $(G, 1)$.

For graph-thoretic notation not defined in this paper, we refer the reader to [2]. Since we deal with multigraphs, we should give the definition of 2-connected graphs in a precise manner. In this paper, a multigraph G is said to be 2-connected if $|V(G)| \geq 2$

and for every pair of distinct vertices, there exist two independent paths between them. Note that, under this definition, a graph consisting of two vertices and two or more edges joining them is 2-connected. For a multigraph G of order at least three, it is easy to see that G is 2-connected if and only if $G - v$ is connected for each $v \in V(G)$.

When we consider a path or a cycle, we always assign a orientation. Let $P = a_0a_1 \dots a_l$ be a path. Then the subpath $a_ia_{i+1} \dots a_{j-1}a_j$ $(i \leq j)$ is denoted by $a_i \overrightarrow{P} a_j$. The same subpath, traversed in the opposite direction, is denoted by $a_j \overleftarrow{P} a_i$. The vertices a_{i+1} and a_{i-1} are denoted by a_i^+ and a_i^-, respectively. For a multigraph G and for $e = xy \in E(G)$, let $V(e)$ denote the set of its endvertices: $V(e) = \{x, y\}$. And for $A \subset V(G)$, let $E(A) = \{e \in E(G) \colon V(e) \subset A\}$.

2 Graphs whose cycles have the same modularity

In this section, we give answers to Problem 2. But before that, we give several lemmas.

Lemma 1 *Let (G, f) be a weighted multigraph in $\mathfrak{C}(a, b)$. Let x and y be distinct vertices in G. If there exist three independent paths P_1, P_2, P_3 from x to y, then $f(P_1) = f(P_2) = f(P_3)$ and $2f(P_1) = a$.*

Proof. Let $C_{12} = x\overrightarrow{P_1}y\overleftarrow{P_2}x$ and $C_{13} = x\overrightarrow{P_1}y\overleftarrow{P_3}x$. Then both C_{12} and C_{13} are cycles, and $f(C_{12}) = f(C_{13}) = a$. On the other hand, $f(C_{12}) = f(P_1) + f(P_2)$ and $f(C_{13}) = f(P_1) + f(P_3)$. Hence $f(P_2) = f(P_3)$. Similarly, we have $f(P_1) = f(P_2)$. Then

$$a = f(C_{12}) = f(P_1) + f(P_2) = 2f(P_1). \qquad \square$$

We also use the following theorem due to Dirac [3].

Theorem B ([3]) *Every simple 2-connected graph with minimum degree at least three has a subdivision of K_4.*

Now we are ready to give an answer to each question in Problem 2.

Theorem 2 *The class $\mathfrak{C}(a, b)$ is nonempty if and only if a is even or b is odd.*

Proof. Suppose $\mathfrak{C}(a,b)$ is nonempty, and let (G,f) be a weighted multigraph in $\mathfrak{C}(a,b)$. We claim that there exist three independent paths P_1, P_2 and P_3 between some pair of vertices x, y in G. Let C be a cycle of G, and let $x \in V(C)$. Since $\deg_G x \geq 3$, there exists an edge $e = xy' \in E(G) - E(C)$ which is incident with x. If $y' \in V(C)$, then xy', $x\overrightarrow{C}y'$ and $x\overleftarrow{C}y'$ are three independent paths between x and y'. If $y' \notin V(C)$, then since G is 2-connected, there exists a path P from y' to some vertex y in $V(C) - \{x\}$ with $V(P) \cap V(C) = \{y\}$ in $G - x$. Then $xy'\overrightarrow{P}y$, $x\overrightarrow{C}y$ and $x\overleftarrow{C}y$ are required three independent paths. Hence the claim follows.

For three indendent paths P_1, P_2 and P_3 between some pair x, y of vertices, we have $2f(P_1) = a$ by Lemma 1. However, the equation $2x = a$ has a solution in $\boldsymbol{Z}/b\boldsymbol{Z}$ only if $a \equiv 0$ or $b \equiv 1 \pmod 2$.

Conversely, let $a \equiv 0 \pmod 2$ or $b \equiv 1 \pmod 2$. Let a' be an element in $\boldsymbol{Z}/b\boldsymbol{Z}$ with $2a' = a$. Let G be the graph of order two in which two vertices in G are joined by triple edges. Then (G, a') belongs to $\mathfrak{C}(a,b)$. $\square$

Theorem 3 *The class $\mathfrak{C}(a,b)$ contains a weighted simple graph if and only if $a = 0$.*

Proof.

First we prove the "only if" part. Suppose (G,f) be a weighted simple graph in $\mathfrak{C}(a,b)$. By Theorem B, G has a subdivision of K_4. Let H be a subgraph of G which is isomorphic to a subdivision of K_4, and let $\{x_1, x_2, x_3, x_4\}$ be the set of the vertices of H that have degree three in H. For distinct indices i, j, we denote by x_iHx_j the unique path from x_i to x_j in H that does not contain any other x_k ($k \in \{1,2,3,4\} - \{i,j\}$). Since x_4Hx_1, $x_4Hx_2Hx_1$ and $x_4Hx_3Hx_1$ are three independent paths in G, by Lemma 1, we have

$$f(x_4Hx_1) = f(x_4Hx_2) + f(x_2Hx_1) = f(x_4Hx_3) + f(x_3Hx_1)$$

and $2f(x_4Hx_1) = a$. By a similar argument, we have $2f(x_4Hx_2) = 2f(x_4Hx_3) = a$. Since the equation $2x = a$ has at most two solutions in $\boldsymbol{Z}/b\boldsymbol{Z}$, we may assume $f(x_4Hx_1) = f(x_4Hx_2)$. Then we have $f(x_2Hx_1) = 0$. Now by considering three independent paths x_1Hx_2, $x_1Hx_3Hx_2$ and $x_1Hx_4Hx_2$, by Lemma 1, we have $a = 2f(x_1Hx_2) = 2 \cdot 0 = 0$.

Since $(G, 0)$ belongs to $\mathfrak{C}(0, b)$ for a 2-connected simple graph G with minimum degree at least three, the "if" part is obvious. $\square$

Finally, we give an answer to the last question of Problem 2

Theorem 4 *The class $\mathfrak{C}(a, b)$ contains a weighted simple graph of the form $(G, 1)$ if and only if $a = 0$ and $b = 2$.*

Because of the existence of a 2-connected simple bipartite graph with minimum degree at least three, the "if" part of the theorem is obvious. We give two proofs to the "only if" part. First one uses Theorem 3.

Proof of Theorem 4. Let $(G, 1)$ be a weighted simple graph in $\mathfrak{C}(a, b)$. By Theorem 3, $a = 0$. Let P be a longest path of G. Let x be the terminal vertex of P. Since $\deg_G x \geq 3$ and P is a longest path, $N_G(x) \subset V(P)$ and $|N_G(x) - \{x^-\}| \geq 2$, and thus we can take u, $v \in N_G(x) - \{x^-\}$ such that $v \in u^+\overrightarrow{P}x$. Then since both $u\overrightarrow{P}xu$ and $v\overrightarrow{P}xv$ are cycles and hence have length congruent to $0 \pmod{b}$, $l(u\overrightarrow{P}x) \equiv l(v\overrightarrow{P}x)$ and hence $l(u\overrightarrow{P}v) \equiv 0 \pmod{b}$. However, this implies that the cycle $u\overrightarrow{P}vxu$ has length congruent to 2 modulo b, and hence $2 \equiv 0 \pmod{b}$. This is possible only if $b = 2$. $\square$

The second proof is deduced immediately from the result by Bondy and Vince [1]. They proved the following theorem.

Theorem C ([1]) *Every 2-connected graph with minimum degree at least three has two cycles C_1 and C_2 with $l(C_1) - l(C_2) \leq 2$.*

From this theorem, we immediately have $2 \equiv 0 \pmod{b}$ for $b \geq 2$, and hence $b = 2$.

3 Edges with weight zero and removable edges

In this section, we study the number of edges with weight zero. In the proof of the "if" part of Theorem 3, we use a weighted graph $(G, 0)$. Therefore, we may suspect that every weighted simple graph (G, f) in $\mathfrak{C}(0, b)$ has an edge e with $f(e) = 0$. However, this is not true if b is even. Let $c = \frac{1}{2}b$, and let G be a 2-connected bipartite graph with minimum degree at least three. Then (G, c) belongs to $\mathfrak{C}(0, b)$.

On the other hand, if b is odd, there exists an edge with weight zero, and the number of edges receiving non-zero weight is bounded from above.

Theorem 5 *Let b be an odd integer with $b \geq 3$, and let (G, f) be a weighted simple graph of order p in $\mathfrak{C}(0, b)$. Then the number of edges e with $f(e) \neq 0$ is at most* $\max\{0, p - 6\}$.

Hereafter, we only consider simple graphs.

The proof of this theorem uses removable edges in 2-connected graphs. Let e be an edge in a 2-connected graph G. Then e is said to be *removable* if $G - e$ is 2-connected. An edge which is not removable is said to be *nonremovable*.

The next lemma gives a relation between edges with non-zero weight and removable edges in 2-connected graphs.

Lemma 6 *Let b be an odd integer with $b \geq 3$, and let (G, f) be a weighted simple graph in $\mathfrak{C}(0, b)$. Then $f(e) = 0$ for each removable edge e in G.*

Proof. Let $e = xy$ be a removable edge of G. Then $G - e$ is 2-connected, and hence there exists two independent paths P_1 and P_2 from x to y in $G - e$. Then P_1, P_2 and e form three independent paths in G, and by Lemma 1, we have $2f(e) = 0$. Since b is odd, we have $f(e) = 0$. $\square$

By Lemma 6, the number of edges with non-zero weight is bounded from above by the number of nonremovable edges.

For $e \in E(G)$ and $S \subset V(G)$, we say that (e, S) is a *separating pair* if $(G - e) - S$

is disconnected. The folloing lemma is trivial but useful paraphrasing of the definition of nonremovable edges.

Lemma 7 *Let G be a 2-connected graph and let $e \in E(G)$. Then e is nonremovable if and only if (e, S) is a separating pair for some S with $|S| = 1$. Furthermore, if (e, S) is a separating pair with $|S| = 1$, then $(G - e) - S$ has exactly two components.*

First, we prove a simple lemma. For a graph G and its edge e, let G/e denote the graph obtained from G by the contraction of e. Note that since we now deal with only simple graphs, if multiple edges arises from the contraction, we replace them with simple edges.

Lemma 8 *Let G be a 2-connected graph of order at least four, and let $e \in E(G)$. Then either $G - e$ is 2-connected or G/e is 2-connected.*

Proof. Assume neither $G - e$ nor G/e is 2-connected. Let $e = xy$. Since G/e is not 2-connected, $G - \{x, y\}$ is disconnected. Let A be a component of $G - \{x, y\}$, and let $\overline{A} = V(G) - (\{x, y\} \cup A)$. Furthermore, since e is not removable, there exists a separating pair $(e, \{z\})$ for some $z \in V(G)$, and $(G - e) - z$ has exactly two components, say B and $\overline{B}$. We may assume $x \in B$ and $y \in \overline{B}$. We may also assume $z \in \overline{A}$ by symmetry. Since $A \neq \emptyset$, again by symmetry, we may assume $A \cap B \neq \emptyset$. However, this implies that $\{x\}$ separates $A \cap B$ and $\overline{A} \cup \overline{B}$, which contradicts the connectivity of G. Hence the lemma follows. $\square$

Now we investigate the number of removable edges in a cycle. For this purpose, we first prove the following lemma.

Lemma 9 *Let G be a 2-connected graph with minimum degree at least three. Let C be a cycle in G and let $F \subset E(C)$. If there exist a separating pair (e, S) with $e \in E(C)$ and $|S| = 1$ and a component A of $(G - e) - S$ such that $E(S \cup A) \cap F = \emptyset$, then there exists a removable edge of G in $E(C) - F$.*

Proof. Assume, to the contrary, that every edge in $E(C) - F$ is nonremovable. Choose a separating pair (e, S) with $e \in E(C)$ and $|S| = 1$, and a component A of

$(G-e)-S$ with $E(S \cup A) \cap F = \emptyset$ so that $|A|$ is as small as possible. Let $e = x^-x$ and $\overline{A} = V(G) - (S \cup A)$. We may assume $x^- \in \overline{A}$ and $x \in A$. Let $f = xx^+$. Then $f \in E(S \cup A)$. Since $E(S \cup A) \cap F = \emptyset$, $f \in E(C) - F$. Thus, by the assumption, f is nonremovable. Then there exists a separating pair (f, T) for some $T \subset V(G)$ with $|T| = 1$. Let B and $\overline{B}$ be the components of $(G-f)-T$. We may assume $x \in B$ and $x^+ \in \overline{B}$. Let

$$U_1 = (S \cap B) \cup (S \cap T) \cup (A \cap T),$$

$$U_2 = (A \cap T) \cup (S \cap T) \cup (S \cap \overline{B}),$$

$$U_3 = (S \cap \overline{B}) \cup (S \cap T) \cup (\overline{A} \cap T), \quad \text{and}$$

$$U_4 = (\overline{A} \cap T) \cup (S \cap T) \cup (S \cap B).$$

Since $x \in A$, $x^+ \in S \cup A$. First, assume $x^+ \in S$. Then $S = \{x^+\}$ and $S \cap T = S \cap B = \emptyset$. If $A \cap T = \emptyset$, then $U_1 = \emptyset$. Since $\delta(G) \geq 3$, $|A \cap B| \geq 2$. Then $\{x\}$ separates $A \cap B - \{x\}$ and $\overline{A} \cup \overline{B}$ in G, which contradicts the connectivity of G. Therefore, we have $A \cap T \neq \emptyset$, which implies $\overline{A} \cap T = \emptyset$ and $x^- \in B$. Then $U_4 = \emptyset$ an hence $G - x^-x$ is disconnected. This again contradicts the connectivity of G.

Next, assume $x^+ \in A$. Then $A \cap B \neq \emptyset$ and $A \cap \overline{B} \neq \emptyset$. Since U_2 separates $A \cap \overline{B}$ and $\overline{A} \cup B$ in $G - xx^+$, $|U_2| \geq 1$. If $|U_2| = 1$, then (f, U_2) is a separating pair and $A \cap \overline{B}$ is a component of $(G-f) - U_2$. Since $U_2 \cup (A \cap \overline{B}) \subset S \cup A$, $E(U_2 \cup (A \cap \overline{B})) \cap F = \emptyset$. Since $A \cap \overline{B} \subsetneq A$, this contradicts the choice of (e, S, A). Therefore, we have $|U_2| \geq 2$. Since $|S| = |T| = 1$, we have $|A \cap T| = |S \cap \overline{B}| = 1$ and $S \cap T = \overline{A} \cap T = S \cap B = \emptyset$. This implies $x^- \in \overline{A} \cap B$ and $U_4 = \emptyset$. Then again $G - x^-x$ is disconnected. This is a contradiction, and the lemma follows. $\square$

Theorem 10 *Let G be a 2-connected graph with minimum degree at least three. Then for every cycle C in G, there exist at least two removable edges of G in C.*

Proof. If all the edges of C are removable, the theorem obviously holds. Suppose C has a nonremovable edge e. Then there exists a separating pair (e, S) for some $S \subset V(G)$ with $|S| = 1$. Let A and $\overline{A}$ be the components of $(G-e)-S$. Apply Lemma 9 with

$F = \emptyset$. Then we have that there exists a removable edge e_1 in $E(C)$. We may assume $e_1 \in E(\overline{A} \cup S)$. The we can apply Lemma 9 again with $F = \{e_1\}$, we see that there exists a removable edge $e_2 \in E(C) - \{e_1\}$. $\square$

Theorem 5 is a consequence of Lemma 6 and the following theorem.

Theorem 11 *Let G be a 2-connected graph of order p with minimum degree at least three. Then G has at most* $\max\{0, p-6\}$ *nonremovable edges.*

Proof. We proceed by induction on p. Assume G has a nonremovable edge $e = xy$. Then $G - e$ is not 2-connected, and $G - e$ has exactly two endblocks, say B_1 and B_2. Let c_i be the unique cutvertex of $G - e$ contained in B_i ($i = 1, 2$). Note that possibly $c_1 = c_2$. Since $\delta(G) \geq 3$, $|B_i| \geq 3$ ($i = 1, 2$). Then $B_i - (\{c_i\} \cup \{x, y\}) \neq \emptyset$, and let $b_i \in B_i - (\{c_i\} \cup \{x, y\})$. Since $\deg_{G-e} b_i \geq 3$, we have $|B_i| \geq 4$ ($i = 1, 2$). This implies

$$|G| \geq |B_1| + |B_2| - |B_1 \cap B_2| \geq 4 + 4 - 1 = 7.$$

Therefore, G has no nonremovable edges if $p \leq 6$, and hence the theorem holds for $p \leq 6$. In particular, G has $p - 6$ nonremovable edges if $p = 6$.

Suppose $p \geq 7$ and G has a nonremovable edge $e = xy$. By Lemma 8, G/e is 2-connected. Let $G' = G/e$. Assume $\delta(G') = 2$. This occurs only if $\deg_G z = 3$ for some $z \in N_G(x) \cap N_G(y)$. Then since $G - e$ is not 2-connected, z is the only cutvertex of $G - e$, and $G - e$ has exactly two blocks, say B_1 and B_2. We may assume $x \in B_1$ and $y \in B_2$. Since $\deg_G z = 3$, we may assume $N_G(z) \cap B_1 = \{x\}$. However, since x cannot be a cutvertex of $G - e$, we have $B_1 = \{x, z\}$. This implies $\deg_G x \leq 2$, a contradiction. Therefore, $\delta(G') \geq 3$.

Since G' is 2-connected and $\delta(G') \geq 3$, by the induction hypothesis, G' has at most $p - 7$ nonremovable edges.

Assume that a nonremovable edge $f = uv$ of G which is different from e becomes removable in G'. Since f is nonremovable in G, $(G - f) - w$ has exactly two components, say A and $\overline{A}$, for some $w \in V(G)$ by Lemma 7. Then f becomes removable in G' only if $A \cup \{w\} = \{x, y\}$ or $\overline{A} \cup \{w\} = \{x, y\}$. But in either case, we have $\deg_G x = 2$ or

$\deg_G y = 2$, a contradiction. Therefore, every nonremovable edge in G except for e is nonremovable in G'.

Assume two distinct nonremovable edges f_1 and f_2 coincide with the same edge in G'. This occurs only if $\{f_1, f_2\} = \{xz, yz\}$ for some $z \in N_G(x) \cap N_G(y)$. However, this implies that $xyzx$ is a cycle in G and that all the edges in this cycle is nonremovable in G. This contradicts Theorem 10. Therefore, every pair of distinct nonremovable edges f_1 and f_2 with $e \notin \{f_1, f_2\}$ are still a pair of distinct nonremovable edges in G'. Thus, considering e, we see that the number of nonremovable edge in G is at most $p - 7 + 1 = p - 6$. $\square$

Both Theorem 5 and Theorem 11 are sharp. Consider the graph G_n defined by

$$\begin{aligned} V(G) &= \{x_i, y_i \colon 0 \le i \le n-1\} \\ E(G) &= \{x_{i-1}x_i, y_{i-1}y_i \colon 1 \le i \le n\} \cup \{x_iy_i \colon 0 \le i \le n-1\} \\ &\quad \cup \{x_0y_1, y_0x_1, x_{n-2}y_{n-1}, y_{n-2}x_{n-1}\}. \end{aligned}$$

Then $|G| = 2n$. We define a weight function f by

$$\begin{aligned} f(x_ix_{i+1}) &= 1 \quad (1 \le i \le n-3) \\ f(y_iy_{i+1}) &= b-1 \quad (1 \le i \le n-3) \\ f(e) &= 0 \quad (\text{all the other edges } e) \end{aligned}$$

Then all the nonremovable edges have non-zero weight, and $f(C) \equiv 0 \pmod b$ for every cycle C in G. Furthermore, the number of nonremovable edges is $2(n-3) = |G| - 6$.

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